Technical Note

Effect of inviscid stagnation flow on the freezing of fluid— a theoretical analysis

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1. Introduction

The problems of phase change from liquid to solid or vice versa have attracted considerable attention in view of both their theoretical interest and their practical applications. Since the work by Stefan [1], many authors have studied the phase-change problems without fluid flow [2,3], and with forced convection [4,5] or natural convection [6,7] in the melt. In most of the works with forced convection, the rate of convective heat transfer from the liquid side of solid–liquid interface was assumed to be known, and the temperature distribution in solid and the location of the solid–liquid interface were determined [4,5]. It is evident, however, that the phase-change process can be affected by the transient development of the temperature distribution in liquid and vice versa. Recently, Yoo [8,9] considered the transient behavior of the temperature distribution in both solid and liquid phases and the freezing rate in the rotating-disk–revolving-fluid system. On the other hand, Rangel and Bian [10,11] investigated an inviscid stagnation-flow solidification problem, and Bian and Rangel [12] studied a viscous stagnation-flow solidification problem. Both of the rotating-disk–revolving-fluid systems and the stagnation-flow solidification problem have forced fluid flows toward the solid–liquid interface, and accordingly many similar characteristics were found.

In this study, we consider the inviscid stagnation-flow solidification problem. The fluid with temperature higher than its freezing temperature flows toward the cold substrate (Fig. 1). Initially ($t = 0$), the fluid is kept at a uniform temperature (T_H) higher than the freezing temperature of fluid (T_F). For $t > 0$, the temperature of the substrate is suddenly lowered to T_C ($T_C < T_F$) and maintained constant. As a result, solidification occurs at the surface of the substrate and the solid grows with time. If there is no fluid flow, then this problem becomes the well-known Stefan problem with Neumann's solution [1]. Recently, Rangel and Bian studied this problem with numerical method [10], and with the method of instantaneous similarity and quasi-steady approximation [11], and showed that the solidification front grows asymptotically to a finite maximum value as time goes to infinity.

We obtain analytic solutions at the initial stage of freezing and the final equilibrium state. The initial-stage solution is obtained by expanding it in powers of time ($\tau \ll 1$), $\theta_{S,L}(\tau, \eta) = \theta_{S,L}^0(\eta) + \theta_{S,L}^1(\eta)\tau + \cdots$ and $\delta^2(\tau) = b_0\tau + b_1\tau^2 + \cdots$, and the final equilibrium state is determined from the steady-state governing equations. In the expression of the temperature distribution and the solid thickness at the initial stage, the terms $\theta_{S,L}^0(\eta)$ and $b_0\tau$ represent the Neumann's solution in the absence of the fluid flow, and the terms $\theta_{S,L}^1(\eta)\tau$ and $b_1\tau^2$ are created by the fluid flow. We can clearly see the effect of the stagnation flow on the pure conduction problem of Neumann [1] from the solutions of the initial stage and the final equilibrium state.

The main physical quantities in the present problem are the growth rate of solid and the heat transfer rate at the surface of the solid and the liquid side of solid–liquid interface. We obtain dimensionless governing

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Nomenclature

A	potential flow strain rate
C_S	specific heat of solid
erf(x)	error function
erfc(x)	complementary error function, $1-\mathrm{erf}(x)$
K	thermal conductivity
K_R	ratio of thermal conductivity, K_S/K_L
L	latent heat
Log(x)	Log$_{10}(x)$
$Q_S(t)$	heat flux at the surface of solid, $-K_S[\partial T_S(y,t)/\partial y]$ at $y=0$
$Q_L(t)$	heat flux at the liquid side of solid–liquid interface, $-K_L[\partial T_L(y,t)/\partial y]$ at $y=X(t)$
Q_{steady}	steady-state heat flux, $Q_S(t)=Q_L(t)=Q_{\mathrm{steady}}$ at $t\rightarrow\infty$
Ste	Stefan number, $C_S(T_F-T_C)/L$
t	time
T	temperature
T_C	cold temperature of solid surface
T_F	freezing temperature of liquid
T_H	hot temperature of liquid
$X(t)$	thickness of solid phase
X_{eq}	solid thickness at equilibrium state
y	axial coordinate

Greek symbols

α	thermal diffusivity
α_R	ratio of thermal diffusivity, α_S/α_L
$\delta(\tau)$	dimensionless solid thickness
δ_{eq}	dimensionless solid thickness at equilibrium state
ζ	dimensionless axial coordinate, $\sqrt{A/\alpha_L}\,y$
η	transformed coordinate, $\zeta/\delta(\tau)$
θ_L	dimensionless temperature in liquid region, $(T_L-T_H)/(T_F-T_H)$
θ_S	dimensionless temperature in solid region, $(T_S-T_C)/(T_F-T_C)$
θ_R	temperature ratio, $(T_H-T_F)/(T_F-T_C)$
ρ	density
σ	growth parameter in Neumann problem
τ	dimensionless time, At

Subscripts

L	liquid
S	solid
∞	infinity

equations which are expressed with three dimensionless parameters of θ_R/K_R, Ste, and α_R, and the characteristics of the solidification process for all the variables are elucidated.

2. Analysis

2.1. Governing equations

We consider the inviscid stagnation flow impinging on a cold substrate (Fig. 1). We assume that the thermophysical properties of solid and liquid phases are constant, and the density change of the material upon freezing is neglected so that there is no fluid flow induced by the volumetric change in the phase-change process. The governing equations are described as follows [11]:

$$\frac{\partial T_S}{\partial t} = \alpha_S \frac{\partial^2 T_S}{\partial y^2} \quad \text{at } 0 < y < X(t) \tag{1}$$
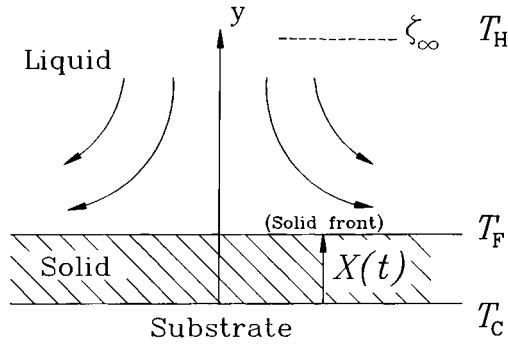
Fig. 1. Stagnation flow ahead of a solidifying interface.

$$\frac{\partial T_L}{\partial t} - 2A[\,y - X(t)]\frac{\partial T_L}{\partial y} = \alpha_L \frac{\partial^2 T_L}{\partial y^2} \quad \text{at } y > X(t). \tag{2}$$

At the solid–liquid interface $y = X(t)$, where the change of state occurs, the energy balance is maintained:

$$K_S \frac{\partial T_S}{\partial y} - K_L \frac{\partial T_L}{\partial y} = \rho L \frac{dX}{dt} \quad \text{at } y = X(t). \tag{3}$$

Additional boundary conditions are

$$T_S = T_C \quad \text{at } y = 0, \quad T_S = T_L = T_F \quad \text{at } y = X(t)$$
$$T_L = T_H \quad \text{at } y \rightarrow \infty. \tag{4}$$

In addition, prior to the onset of solidification, the temperature of liquid is uniform and is equal to $T_H(> T_F)$. The heat fluxes at the surface of solid (Q_S) and the liquid side of solid–liquid interface (Q_L) are defined as

$$Q_S(t) = -K_S \left[\frac{\partial T_S(y,t)}{\partial y} \right] \quad \text{at } y = 0 \tag{5}$$

$$Q_L(t) = -K_L \left[\frac{\partial T_L(y,t)}{\partial y} \right] \quad \text{at } y = X(t) \tag{6}$$

$Q_S(t)$ and $Q_L(t)$ are of the same value, $Q_S(t) = Q_L(t) = Q_{steady}$, at $t \rightarrow \infty$.

Introducing the following dimensionless variables

$$\theta_S = \frac{T_S - T_C}{T_F - T_C}, \quad \theta_L = \frac{T_L - T_H}{T_F - T_H}, \quad \tau = At, \quad \zeta = \sqrt{\frac{A}{\alpha_L}}\, y$$

$$K_R = K_S/K_L, \quad \alpha_R = \alpha_S/\alpha_L, \quad \theta_R = \frac{T_H - T_F}{T_F - T_C},$$
$$Ste = \frac{C_S(T_F - T_C)}{L} \tag{7}$$

governing Eqs. (1)–(4) are written as

$$\frac{\partial \theta_S}{\partial \tau} = \alpha_R \frac{\partial^2 \theta_S}{\partial \zeta^2} \quad \text{at } 0 < \zeta < \delta \tag{8}$$

$$\frac{\partial \theta_L}{\partial \tau} - 2(\zeta - \delta)\frac{\partial \theta_L}{\partial \zeta} = \frac{\partial^2 \theta_L}{\partial \zeta^2} \quad \text{at } \zeta > \delta \tag{9}$$

$$\frac{\partial \theta_S}{\partial \zeta} + \frac{\theta_R}{K_R}\frac{\partial \theta_L}{\partial \zeta} = \frac{1}{\alpha_R\, Ste}\frac{d\delta}{d\tau} \quad \text{at } \zeta = \delta \tag{10}$$

$$\theta_S = 0 \quad \text{at } \zeta = 0, \quad \theta_S = \theta_L = 1 \quad \text{at } \zeta = \delta(\tau),$$
$$\theta_L = 0 \quad \text{at } \zeta \rightarrow \infty \tag{11}$$

where

$$\delta(\tau) = \sqrt{\frac{A}{\alpha_L}}\, X(t) \tag{12}$$

denotes the dimensionless solid thickness. It is assumed that $\delta(0) = 0$.

It is to be noted that θ_R/K_R is one parameter in the dimensionless governing Eqs. (8)–(11). It is because the heat transfer rate is determined by Fourier's law of heat conduction.

Let us introduce the following coordinate transformation to fix the moving boundary of the position of solid–liquid interface $\zeta = \delta(\tau)$ at $\eta = 1$.

$$\eta = \frac{\zeta}{\delta(\tau)}. \tag{13}$$

Eqs. (8)–(11) are transformed as

$$\delta^2 \frac{\partial \theta_S}{\partial \tau} - \frac{\eta}{2}\frac{d\delta^2}{d\tau}\frac{\partial \theta_S}{\partial \eta} = \alpha_R \frac{\partial^2 \theta_S}{\partial \eta^2} \quad \text{at } 0 < \eta < 1 \tag{14}$$

$$\delta^2 \frac{\partial \theta_L}{\partial \tau} - \frac{\eta}{2}\frac{d\delta^2}{d\tau}\frac{\partial \theta_L}{\partial \eta} - 2(\eta - 1)\delta^2\frac{\partial \theta_L}{\partial \eta} = \frac{\partial^2 \theta_L}{\partial \eta^2} \quad \text{at } \eta > 1 \tag{15}$$

$$\frac{\partial \theta_S}{\partial \eta} + \frac{\theta_R}{K_R}\frac{\partial \theta_L}{\partial \eta} = \frac{1}{2\alpha_R\, Ste}\frac{d\delta^2}{d\tau} \quad \text{at } \eta = 1 \tag{16}$$

$$\theta_S = 0 \quad \text{at } \eta = 0, \quad \theta_S = \theta_L = 1 \quad \text{at } \eta = 1,$$
$$\theta_L = 0 \quad \text{at } \eta \rightarrow \infty. \tag{17}$$

2.2. Initial stage of freezing

At the initial stage of freezing ($\tau \ll 1$), $\theta_{S,L}(\tau, \eta)$ and $\delta(\tau)$ can be expanded in powers of time [8]:

$$\theta_{S,L}(\tau, \eta) = \theta_{S,L}^0(\eta) + \theta_{S,L}^1(\eta)\tau + \cdots \tag{18}$$

$$\delta^2(\tau) = b_0\tau + b_1\tau^2 + \cdots. \tag{19}$$

Substitution of Eqs. (18)–(19) into Eqs. (14)–(17) yields the equations for $O(\tau^0)$

$$\frac{d^2\theta_S^0}{d\eta^2} + \frac{b_0}{2\alpha_R}\eta\frac{d\theta_S^0}{d\eta} = 0 \quad \text{at } 0<\eta<1 \tag{20}$$

$$\frac{d^2\theta_L^0}{d\eta^2} + \frac{b_0}{2}\eta\frac{d\theta_L^0}{d\eta} = 0 \quad \text{at } \eta > 1 \tag{21}$$

$$\frac{d\theta_S^0}{d\eta} + \frac{\theta_R}{K_R}\frac{d\theta_L^0}{d\eta} = \frac{b_0}{2\alpha_R Ste} \quad \text{at } \eta = 1 \tag{22}$$

$$\theta_S^0(0) = 0, \quad \theta_S^0(1) = \theta_L^0(1) = 1, \quad \theta_L^0(\infty) = 0 \tag{23}$$

and $O(\tau)$ equations

$$\frac{d^2\theta_S^1}{d\eta^2} + \frac{b_0}{2\alpha_R}\eta\frac{d\theta_S^1}{d\eta} - \frac{b_0}{\alpha_R}\theta_S^1 = -\frac{b_1}{\alpha_R}\eta\frac{d\theta_S^0}{d\eta} \quad \text{at } 0<\eta<1 \tag{24}$$

$$\frac{d^2\theta_L^1}{d\eta^2} + \frac{b_0}{2}\eta\frac{d\theta_L^1}{d\eta} - b_0\theta_L^1 = -[2(\eta-1)b_0 + b_1\eta]\frac{d\theta_L^0}{d\eta} \tag{25}$$

$$\text{at } \eta > 1$$

$$\frac{d\theta_S^1}{d\eta} + \frac{\theta_R}{K_R}\frac{d\theta_L^1}{d\eta} = \frac{b_1}{\alpha_R Ste} \quad \text{at } \eta = 1 \tag{26}$$

$$\theta_S^1(0) = \theta_S^1(1) = \theta_L^1(1) = \theta_L^1(\infty) = 0. \tag{27}$$

The solution of Eqs. (20)–(23) for $O(\tau^0)$ is the Neumann's solution [1].

$$\theta_S^0(\eta) = \frac{\text{erf}(\sigma\eta)}{\text{erf}(\sigma)} \tag{28}$$

$$\theta_L^0(\eta) = \frac{\text{erfc}(\sigma\sqrt{\alpha_R}\eta)}{\text{erfc}(\sigma\sqrt{\alpha_R})} \tag{29}$$

$$\frac{\exp(-\sigma^2)}{\text{erf}(\sigma)} - \frac{\theta_R\sqrt{\alpha_R}}{K_R}\frac{\exp(-\sigma^2\alpha_R)}{\text{erfc}(\sigma\sqrt{\alpha_R})} = \frac{\sqrt{\pi}\sigma}{Ste} \tag{30}$$

$$b_0 = 4\sigma^2\alpha_R. \tag{31}$$

The solution of Eqs. (24)–(27) is found with a homogeneous solution of the form

$$u_S(\eta) = \eta^2 + \frac{1}{2\sigma^2} \tag{32}$$

$$u_L(\eta) = \eta^2 + \frac{1}{2\sigma^2\alpha_R}. \tag{33}$$

The solution is

$$\theta_S^1(\eta) = -u_S(\eta)\int_\eta^1 \frac{b_1 C_1\left(\frac{\eta^4}{4} + \frac{\eta^2}{4\sigma^2}\right) + C_5}{\exp(\sigma^2\eta^2)u_S^2(\eta)} d\eta \quad \text{at} \tag{34}$$

$$0 \le \eta \le 1$$

$$\theta_L^1(\eta) = u_L(\eta)\int_1^\eta \frac{C_3 f(\eta) + b_1 C_2\left(\frac{\eta^4}{4} + \frac{\eta^2}{4\sigma^2\alpha_R}\right) + C_4}{\exp(\sigma^2\alpha_R\eta^2)u_L^2(\eta)} d\eta$$

$$\text{at} \quad \eta \ge 1 \tag{35}$$

where

$$C_1 = -\frac{2\sigma}{\sqrt{\pi}\alpha_R \text{ erf}(\sigma)}, \quad C_2 = \frac{2\sigma\sqrt{\alpha_R}}{\sqrt{\pi} \text{ erfc}(\sigma\sqrt{\alpha_R})}$$

$$C_3 = \frac{16\sigma^3\alpha_R^{3/2}}{\sqrt{\pi} \text{ erfc}(\sigma\sqrt{\alpha_R})},$$

$$f(\eta) = \frac{\eta^4}{4} - \frac{\eta^3}{3} + \frac{\eta^2}{4\sigma^2\alpha_R} - \frac{\eta}{2\sigma^2\alpha_R}.$$

The constants b_1, C_4, and C_5 are determined explicitly by the boundary condition (26), $\theta_S^1(0)=0$, and $\theta_L^1(\infty)=0$. The processes are straightforward and the detailed equations are omitted for brevity.

2.3. Final equilibrium state

In the Neumann problem without fluid flow, the solid grows continuously with time according to the relation $X(t) = 2\sigma\sqrt{\alpha_S t}$. In the present problem, however, the forced fluid flow toward the solid–liquid interface restricts the propagation of the thermal boundary $(T = T_H)$ in the liquid, and consequently the system approaches a final equilibrium state as time goes on. Eqs. (8)–(11) with $\partial/\partial\tau = 0$ yield the following equilibrium state:

$$\theta_S(\zeta) = \frac{\zeta}{\delta_{eq}}, \quad \theta_L(\zeta) = \text{erfc}(\zeta - \delta_{eq}),$$

$$\delta_{eq} = \frac{\sqrt{\pi}}{2}\frac{K_R}{\theta_R}. \tag{36}$$
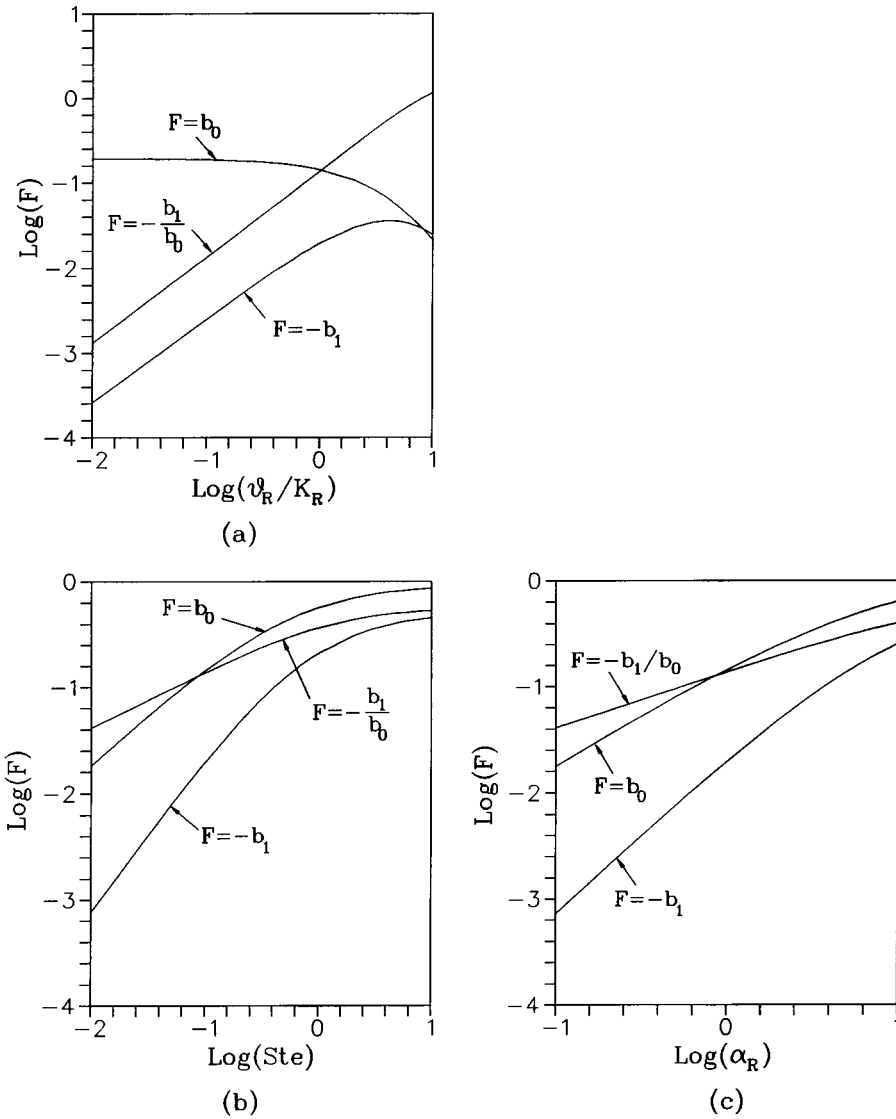
The steady-state heat flux is given by

Fig. 2. Effect of several parameters on the growth of solid, $\delta^2(\tau) = b_0\tau[1 + (b_1/b_0)\tau]$ for $\tau \ll 1$: (a) effect of temperature ratio and conductivity ratio with $\alpha_R = 1$ and $Ste = 0.1$; (b) effect of Stefan number with $\alpha_R = \theta_R/K_R = 1$; (c) effect of diffusivity ratio with $\theta_R/K_R = 1$ and $Ste = 0.1$.

$$Q_{steady} = -K_S \frac{(T_F - T_C)}{X_{eq}} \quad \text{at } t \to \infty \tag{37}$$

and the instantaneous heat fluxes at the surface of solid and the liquid side of solid–liquid interface are expressed as the following equations:

$$\frac{Q_S(At)}{Q_{steady}} = \frac{\delta_{eq}}{\delta(\tau)} \frac{\partial \theta_S(\eta)}{\partial \eta} \quad \text{at } \eta = 0 \tag{38}$$

$$\frac{Q_L(At)}{Q_{steady}} = -\frac{\sqrt{\pi}}{2\delta(\tau)} \frac{\partial \theta_L(\eta)}{\partial \eta} \quad \text{at } \eta = 1. \tag{39}$$

3. Results and discussion

Once the temperature of the substrate has been lowered to $T_C(T_C < T_F)$, the solid grows continuously with time, and approaches a final equilibrium state.
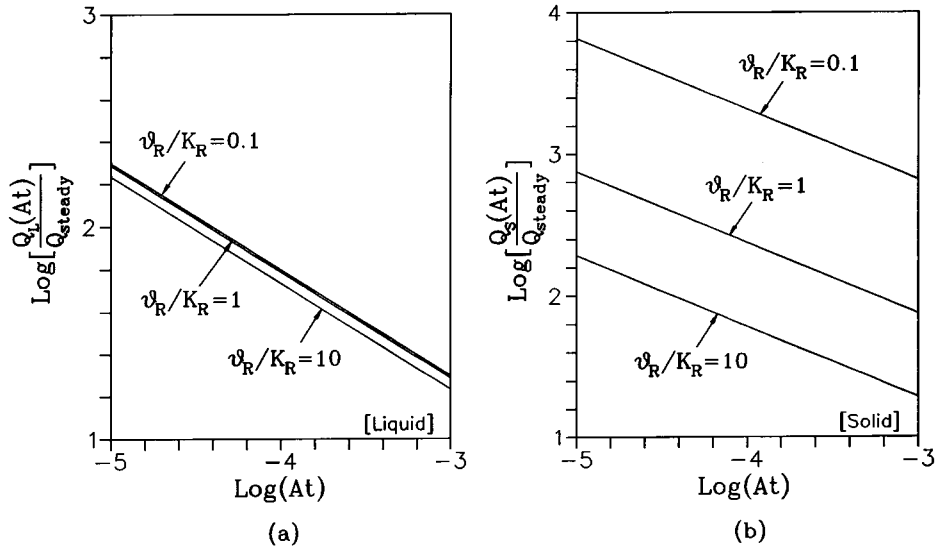
Fig. 3. Transient heat fluxes at the surface of solid (Q_S) and the liquid side of solid–liquid interface (Q_L) for several values of θ_R/K_R with $\alpha_R = 1$ and $Ste = 0.1$: (a) $Q_L(At)/Q_{steady}$; (b) $Q_S(At)/Q_{steady}$.

The thickness of the solid at the initial stage of solidification is approximated as $\delta^2(\tau) = b_0\tau + b_1\tau^2$, and the equilibrium value is obtained as $\delta_{eq} = (K_R/\theta_R)\sqrt{\pi}/2$. We can see the effect of the fluid flow and several parameters, θ_R/K_R, Ste, and α_R, on the solidification process from the solution of the initial stage and the final equilibrium state, since $\delta(\tau)$ increases monotonously

with time, and the initial stage is smoothly connected to the final state.

In the expression of the initial-stage thickness of solid, $\delta^2(\tau) = b_0\tau + b_1\tau^2$, the first term represents the pure conduction solution of Neumann and the second is created by the fluid flow. Calculation shows that b_1 has negative values in all cases, which reveals that the
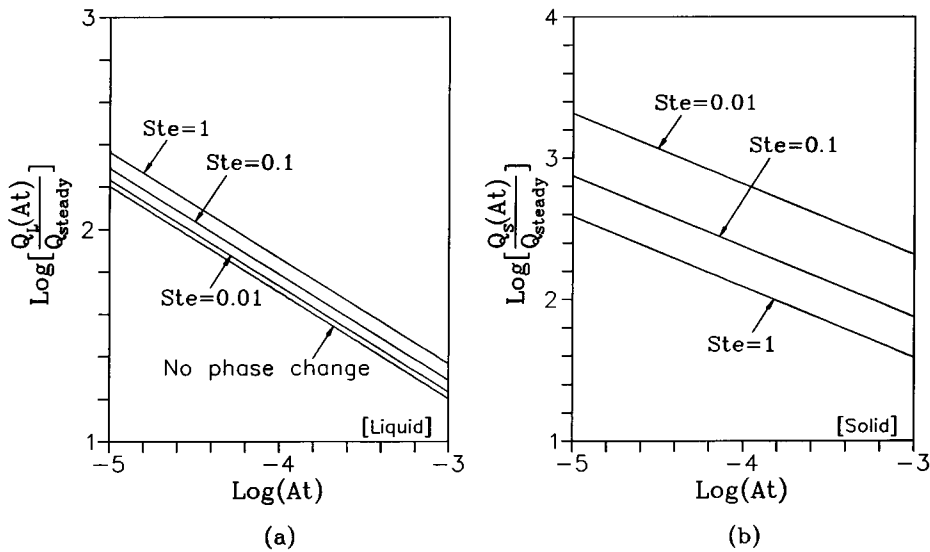

Fig. 4. Transient heat fluxes at the surface of solid (Q_S) and the liquid side of solid–liquid interface (Q_L) for several Stefan numbers with $\alpha_R = 1$ and $\theta_R/K_R = 1$, and Q_L for the case with no phase change in the inviscid stagnation flow: (a) $Q_L(At)/Q_{steady}$; (b) $Q_S(At)/Q_{steady}$.

fluid flow toward the substrate always inhibits the solidification process. The effect of the fluid flow on the growth of solid can be seen with the expression of

$$X^2(t) = 4\sigma^2 \alpha_S t[1 - |b_1/b_0|(At)] \tag{40}$$

in which the term $-|b_1/b_0|(At)$ represents the magnitude of the effect of the fluid flow on the pure conduction problem. The values of b_0, $-b_1$ and $-(b_1/b_0)$ as functions of θ_R/K_R, Ste, and α_R are presented in Fig. 2. Fig. 2(a) represents the effect of the temperature ratio and the conductivity ratio on the growth rate of solid. As θ_R/K_R is increased, the value of b_0 decreases; that is, the growth rate of solid in the pure conduction state is decreased. However, the value of $-(b_1/b_0)$ is increased with increase of θ_R/K_R, which shows that the effect of the fluid flow becomes strong as the temperature (T_H) or conductivity (K_L) of liquid is increased. For small Stefan number of $Ste < 0.1$, the magnitude of $-(b_1/b_0)$ has been observed to be linearly proportional to θ_R/K_R [Fig. 2(a)]: $-(b_1/b_0) \approx C(\theta_R/K_R)$. The approximate values of the constant C at $\alpha_R = 1$ are 0.0127, 0.0409 and 0.133 for $Ste = 0.001$, 0.01 and 0.1, respectively, i.e. C increases with increase of Ste. For the parameters of Ste and α_R, both of b_0 and $-(b_1/b_0)$ are increased with increase of Ste or α_R [Fig. 2(b,c)]. We can see that the effect of the fluid flow on the growth rate of solid is increased as θ_R/K_R, Ste, α_R, or the potential-flow strain rate A becomes large.

Fig. 3 shows the transient heat fluxes at the surface of the solid and the liquid side of the solid–liquid interface for $\theta_R/K_R = 0.1$, 1 and 10. As θ_R/K_R is increased, both of $Q_S(\tau)/Q_{steady}$ and $Q_L(\tau)/Q_{steady}$ are decreased, that is the response time of heat transfer in both solid and liquid phases is decreased, since the maximum solid length (δ_{eq}) that can be grown is inversely proportional to θ_R/K_R.

As shown by $\delta_{eq} = (K_R/\theta_R)\sqrt{\pi}/2$, the equilibrium state is independent of Ste, but the solid grows faster for larger Ste. Fig. 4(a) represents $Q_L(\tau)/Q_{steady}$ for $Ste = 0.01$, 0.1, 1, and the case with no phase change. The unsteady stagnation point heat transfer for viscous fluid without phase change was considered at the early years [13,14]. When phase change is present, $Q_L(\tau)/Q_{steady}$ is larger than that of the case with no phase change, and the difference is increased, as Ste increases. When the temperature of the solid is suddenly lowered to T_C, there is propagation of a thermal boundary from the solid to the liquid region, but the fluid flow toward the solid inhibits the propagation. If the solid grows more rapidly, the propagation of the thermal boundary is more strongly inhibited. Consequently, it tends to restrict the speed of approach to equilibrium state. On the other hand, Fig. 4(b) shows that $Q_S(\tau)/Q_{steady}$ is decreased as Ste is increased, since the growth rate of solid is increased

with increase of Ste, and accordingly the response time is decreased.

The equilibrium state is also independent of α_R [Eq. (36)], and the solid grows faster for larger α_R, as the case of Ste. And accordingly, the dependency of the heat fluxes on the variable α_R shows the same characteristics as that for Ste. It has been observed that the unsteady heat fluxes $Q_S(\tau)$ and $Q_L(\tau)$ are almost unvarying for the changes in α_R and Ste, if α_R Ste = constant; i.e., they are approximately functions of the parameter α_R Ste.

Rangel and Bian [11] studied the inviscid stagnation-flow solidification problem with the method of instantaneous similarity and quasi-steady approximation, and made parametric study with the four variables, Ste, θ_R, $1/\alpha_R$, and $1/K_R$. From the instantaneous similarity solution valid for small time, they obtained the solid thickness as $X(t) = \lambda\sqrt{\alpha_S t}$, where λ was determined numerically for each value of time. They calculated λ's for given values of time (τ), and showed that λ decreased as τ increased. Comparing it with the present solution (40), λ corresponds to $2\sigma\sqrt{1 - |b_1/b_0|\tau}$. The fact that λ is a decreasing function of τ can be easily seen from the expression of $\lambda(\tau) = 2\sigma\sqrt{1 - |b_1/b_0|\tau}$, and the calculated values of λ's for the parameters, Ste, θ_R, $1/\alpha_R$, and $1/K_R$ in [11], with a small time of $\tau = 0.1$ showed good agreement with the results presented by them [11]. For large value of time ($\tau \to \infty$), Rangel and Bian [11] concluded that the temperature distribution and the solid thickness are independent of Ste, while changes in the parameters θ_R, $1/\alpha_R$ and $1/K_R$ affect the long time behavior of the solution. However, the steady-state solution of Eq. (36) shows that the equilibrium state is a function of only θ_R/K_R, but is independent of Ste and α_R. The variable α_R, as well as Ste, is time-governing parameter of the solidifying process, but does not affect the final equilibrium state.

From the observed results presented above, we can see that the parametric study for the inviscid stagnation-flow solidification problem can be made effectively with only the two variables of θ_R/K_R and Ste, although four-dimensionless variables, Ste, θ_R, α_R, and K_R, appear in the problem. This result would help the parametric study in the numerical computation and the other problems with different boundary or initial conditions.

4. Summary

We consider the problem of phase change from liquid to solid in the inviscid stagnation flow. The dimensionless governing equations have three dimensionless parameters of θ_R/K_R, Ste and α_R. The solution at the initial stage of freezing is obtained by expanding

it in powers of time. The equilibrium state is dependent on θ_R/K_R, but is independent of *Ste* and α_R. The effect of the fluid flow on the pure conduction problem can be clearly seen from the solution of the initial stage and the final equilibrium state, and the characteristics of the solidification process for all the dimensionless parameters are elucidated.

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